R300 – Advanced Econometric Methods

PROBLEM SET 4 - QUESTIONS

Due by Mon. November 9

1. Suppose that

$$y_i = x_i\beta_1 + z_i\beta_2 + \varepsilon_i.$$

State and show the Frisch-Waugh-Lovell theorem (say, for β_1) by explicit calculation.

2. Suppose x is continuous and uniformly distributed on the interval $[\theta, \theta + 1]$. We wish to test

$$H_0 : \theta = 0 \quad \text{vs} \quad H_1 : \theta > 0.$$

Consider the procedure

$$\text{Reject } H_0 \text{ if } x > .95, \qquad \text{Accept } H_0 \text{ otherwise.}$$

(i) Compute the size of this test.

(ii) Derive the power function.

3. Suppose $x \sim N(\mu, \sigma^2)$. Consider two independent random samples on x, $\{x_{1i}\}_{i=1}^n$ and $\{x_{2i}\}_{i=1}^n$. Find a sample size n so that

$$P\left(|\bar{x}_1 - \bar{x}_2| < \frac{\sigma}{5}\right)$$

is .99. Explain how you proceed.

4. Suppose that x_i is exponential with with density

$$f_\theta(x) = \theta e^{-x\theta},$$

where $\theta > 0$ and $x \geq 0$.

(i) Derive the maximum likelihood estimator of θ, say $\hat{\theta}$.

(ii) Derive the asymptotic distribution of $\hat{\theta}$.

(iii) Derive the asymptotic distribution of the estimator $1/\hat{\theta}$.

[1]